The Annual Percentage Rate - Complexity and Ambiguity

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Abstract: The annual percentage rate should enable consumers to better evaluate the favourability of a loan and indicates the percentage of the loan which has to be redeemed within one year, considering instalments, maintenance and other charges that go with the loan. In addition, loan providers are obliged, according to the Czech law, to present this rate with their consumer credit offers. This paper outlines the calculation of the annual percentage rate and points out its ambiguity illustrated with concrete examples which depict its weakness and inconsistencies which may cause a dispute between consumers, loan providers and state inspection authorities. All the presented deficiencies lead to the conclusion that the annual percentage rate is not a suitable tool which should be used by financial institutions to help consumers to evaluate the favourability of a loan.

Key words: annual percentage rate · loan

JEL Classification: G23

1 Introduction

The annual percentage rate (APR) should enable consumers to better evaluate the favourability of a loan and indicates the percentage of the loan which has to be redeemed within one year, considering instalments, maintenance and other charges that go with the loan. In addition, loan providers are obliged, according to law, to present this rate with their loan offers. Recently, we could witness some disputes about APR calculation between the Czech Trade Inspection Authority and a non-bank financial institution offering consumer loans using APR calculation methods.

2 Material and methods

The main aim of the paper is to illustrate with concrete examples the weaknesses and inconsistencies of the APR which may cause disputes between consumers, loan providers and state inspection authorities. The concrete examples are calculated in accordance with the formula presented in the Act No. 145/2010 Col. about consumer loans.

2.1 The annual percentage rate definition

In the European Union, the focus of APR standardization is heavily on transparency and consumer rights: “a comprehensible set of information to be given to consumers in good time before the contract is concluded and also as part of the credit agreement [...] every creditor has to use this form when marketing a consumer credit in any member state”. So marketing different figures is not allowed. The EU regulations were reinforced with directives 2008/48/EC and 2011/90/EU (www.ec.europa.eu), fully in force in all member states since 2013.

In the Czech Republic, the APR is, according to Act No. 145/2010 Col. about consumer loans (hereafter the Act), calculated as follows (http://www.zakonyprolidi.cz):

\[
\sum_{k=1}^{m} C_k \cdot (1 + X)^{-t_k} = \sum_{l=1}^{m'} D_l \cdot (1 + X)^{-s_l}
\]

where

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• $X$ is the APR,
• $m$ is the number of the last draw,
• $k$ is the number of the draw (1 ≤ $k$ ≤ $m$),
• $C_k$ is the amount of the draw $k$,
• $t_i$ is an interval expressed in years and fractions of a year between the date of the first draw and the dates of the following draws ($t_f = 0$),
• $m'$ is the number of the last instalment or charge,
• $l$ is the number of a draw of an instalment,
• $D_l$ is the amount of the instalments or charges,
• $s_l$ is an interval expressed in years and fractions of a year between the date of the first draw and the date of the following instalments or charges.

2.2 The unequivocality of the annual percentage rate

If the APR should enable us to compare loans with different parameters, it must have the following attributes:

• The APR must exist for every loan.
• The value of the APR must be unequivocally defined; there must be exactly one solution of the APR equation.
• The calculation of the time periods should reflect common conventions.
• The rule for rounding must be defined unequivocally.

It should be mentioned that the calculation of the roots of the given equation is not trivial and it is possible, in most cases, only with the use of a computer. It is important to keep in mind that if any of the above mentioned attributes are not met, the APR cannot fulfil to its purpose.

3 Results

It is not a problem to make an example of a loan for which there is no value or more than one value of the APR calculated in accordance with the Act – see the examples below. Generally speaking, we may say that the given definition (equation) may have up to $n$ different roots (solutions), where $n$ is the number of periods where there is a cash-flow. This is a fact which has not been considered properly and which does not correspond with the nature of the Act. Moreover, some roots may even be negative and the negative APR has no economic interpretation.

Example 1: We ask a bank for a loan of 100,000 CZK. First, we have to pay a single approval charge of 14,000 CZK, and after 3 months, we get the loan which is redeemable by the amount of 110,000 CZK in 3 years later.

The corresponding APR is calculated as follows:

$$100000 \cdot (1 + X)^0 = 14000 \cdot (1 + X)^{\frac{3}{12}} + 110000 \cdot (1 + X)^{-\frac{33}{12}}$$

Figure 1 Graphical solutions of Example 1

Source: authors
As is possible to observe from Fig. 1, there are two solutions $X_1 = 0.0951$ and $X_2 = 2602$, i.e. the corresponding APRs are 9.51% and 260.200%. There is no doubt that the second value of the ARP is nonsense.

**Example 2:** We ask a bank for a loan of 100,000 CZK. First, we have to pay a single approval charge of 80,000 CZK, and after 3 months, we get the loan which is redeemable by the amount of 60,000 CZK in 3 years later.

The corresponding APR is calculated as follows:

\[
100000 \cdot (1 + X)^0 = 80000 \cdot (1 + X)^{\frac{3}{12}} + 60000 \cdot (1 + X)^{\frac{33}{12}}
\]

**Figure 2** Graphical solution of Example 1

As it is possible to observe from Fig. 2, there is no solution and therefore there is no corresponding APR.

A problem may be identified with the length of the interest period. In banking and financial mathematics, there are three basic and standard methods of calculation the interest period – the European standard (\(\frac{360}{360}\)), banker’s interest (\(\frac{ACT}{360}\)) and exact interest (\(\frac{ACT}{ACT}\)). The European standard considers every month to have exactly 30 days, the banker’s and the exact interests consider months with different lengths. However, the Act calculates the length of every month to have \(\frac{365}{12}\) days, even when the year is a leap year. It may cause problems when various financial products use different methods of interest period calculation.

Appendix 5d) of the Act says that “the root of the equation $X$ has to be rounded to at least one decimal place. If the digit on the following decimal place is equal to or bigger than 5, the digit at the given decimal place is raised by 1.” It means that, for example, for $X = 0.2315$ after rounding we get 0.2, i.e. the ARP = 20%. However, for both $X = 0.051$ and $X = 0.149$ we get after rounding the same ARP = 10%. This is obviously nonsense and it may be considered as deceiving of consumers.

Some financial institutions offer loans to their customers only if they pay a kind of loan insurance on the ability to repay the given loan. This causes another problem, because then the insurance premium is an obligatory charge going with the loan. However, the consumers do not pay back in their instalments only the loan principle, the interest and other charges but they pay for an extra service – the insurance. Should or should not the insurance be included in the APR? Unfortunately, this fact is not treated by the Act.

We can also encounter a case when a non-bank financial company offers a consumer a product and arranges the loan itself with a third party for the consumer. The mediation charge could be seen again as a kind of service and again, there is the question of whether the charge should be included in the APR calculation, as the Act considers only two parties. This situation is illustrated by Examples 3 and 4.

**Example 3:** A non-banking financial company offers a product worth 100,000 CZK. The loan of 100,000 CZK is provided by a bank of the same financial group and there is a mediation charge of 8,000 CZK. The total principal (the loan and the mediation charge) is redeemed with 84 monthly instalments of 2,464 CZK.
The client has to repay the total amount of 108,000 CZK with monthly instalments of 2,464 CZK for 84 months. The appropriate APR is 26.7%.

**Example 4:** A non-banking financial company offers a product worth 60,000 CZK. The loan of 60,000 CZK is provided by a bank of the same financial group and there is a mediation charge of 48,000 CZK. The total principal (the loan and the mediation charge) is redeemed with 84 monthly instalments of 2,464 CZK.

The client has to repay the total amount of 108,000 CZK with monthly instalments of 2,464 CZK for 84 months. The appropriate APR is 51.3%.

As we can see, even though the client repays, in both cases, the same instalment for the same time period, the APRs differ. Another problem can be encountered when a bank requires, for example in the case of a mortgage, that the client has to effect mortgage indemnity insurance which is included in monthly instalments. Then, again, it is an extra service and it is disputable if to include this extra service in the APR calculation or not.

4 Conclusion

The aim of this paper was to outline the complexity of the APR calculation, discuss its characteristics embedded in the Act and to point out a certain ambiguity which may cause a dispute between consumers, loan providers and state inspection authorities. When considering all the mentioned deficiencies, it seems that the APR is not a suitable tool which should be used by financial institutions to help consumers to evaluate the favourability of a consumer credit. The evaluation of credit offers should be considered as a problem of great complexity and to compare various credit offers using only one aspect seems to be rather defective.

References
