Probabilistic Optimization in Environmental Politics

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Abstract: In this paper we provide an optimization methodology to deal with problems of incorporating ecological arrangements to new big industrial or transport constructions. The methodology relies on stochastic optimization, namely optimization with probabilistic (chance) constraints. We describe main features of the model, identify an uncertainty factors, and formulate the problem as a problem with stochastic constraints. To illustrate the introduced methodology we provide a simple example involving indicators of ecological stability.

Key words: Indicators of Ecological Stability · Uncertainty · Stochastic Optimization · Probabilistic Constraints

JEL Classification: C44 · C61

1 Introduction: Environmental Policy

The modern society is characterized, among others, by a growing number of constructions as a result of public and private investments that support economic development. Many of these investments are expected to create and support the environment for subsequent private sector investments, as investments into education, research, innovations, science, labor market. Our interest is devoted to the building activities: preparation of industrial zones around big cities and necessary additional infrastructure as represented for example by transport line constructions (highways and railways). The transportation strategy policy, as defined by the Government of the Czech Republic, covers several main purposes:

- construction of motorways and expressways;
- construction of municipality road by-passes;
- modernization of international roads;
- increase of traffic safety;
- quality improvement of the roads.

Similar policies are defined on the government, regional, or municipal basis for other industrial development. This is not a new phenomenon – such policies accompany the human activities for long times. What is new, compared to thirty years old standards, is an emphasis on the maximal thriftiness of the activities to the environment. This is of course motivated by positive impacts of such behavior, as the overall protection of the environment, sustainable use of the natural resources, a reduction of the environmental load, or an enhancement of the quality of the life. This requires a very distinct approach to be held already at time of planning.

Nowadays, any new big construction cannot be realized without precise treatment of impacts to the environment (negative as well as positive ones). In European Union, this is required by the Council Directive 85/337/EEC of 27 June 1985, on the assessment of the effects of certain public and private projects on the environment, with some later complements, and implemented by national rules. The process is divided into several phases; the most important one is the so-called Environmental Impact Assessment (EIA). The main purpose is to evaluate the (industrial or transport) construction in order to identify its negative impacts on the environments, to state if these impacts are acceptable (possibly compensated by positive benefits of the construction), and to propose some obligatory arrangements to diminish the load of the environment caused by the construction.

As an example, in Houda (2010) we presented an application of EIA to a highway construction and described thoroughly each of EIA categories applied to such construction. These categories are divided usually into two main classes:

- influences of the construction to the human healthy, and
- influences of the construction to the environment.

The first class covers air pollution factors, noise pollution, and social-economic (“comfort”) factors. The second class covers air and climate impacts, water impacts, impacts on land and forests, impact on mineral and natural resources, impact on flora, fauna and ecosystems, impact on landscape, impact on systems of ecological stability, and impact on tangible property and cultural heritage. Every such class covers many inputs and outputs, some of them are

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very hardly quantifiable (like emotional factors or life conditions in the category of social-economic factors). Furthermore, many sources of uncertain nature can be identified, for example:

- future load of transport networks (traffic),
- efficiency of the proposed arrangements,
- subjective criteria,
- unpredictable accidents.

In our research we propose a quantitative methodology to deal with environmental tasks; it takes use of known modelling frameworks of stochastic optimization, namely probabilistic optimization methods.

2 Model Description

Consider a collection of possible arrangements which can be used to compensate for negative environmental impact of a construction. According to common conventions, the collection will be denoted $X \subset \mathbb{R}^n$ and the arrangements (elements of $X$) by $x$. The nature of the components of $x$ can be very manifold—from 0–1 variables (representing just on/off state of the arrangements) through discrete/integer values (variants of arrangements, equivalence and exclusion constraints), to the continuous variables (describing dimensions and quantities of the arrangements). The number of the variables (that is, $n$) and their nature will result without doubts in numerical limitations and would require some additional research in order to simplify the representation of the economic reality into a reasonable sized mathematical model.

A traditional representation of the uncertainty is through a variable $\xi \in \Xi$. The support $\Xi \subset \mathbb{R}^n$ is usually referred as the uncertainty set, and is assumed to be fixed. In robust optimization, we generally do not require any additional information about $\xi$; the situation is different in our stochastic optimization approach which solely depends on complete knowledge of the probabilistic distribution of $\xi$—we assume that $\xi$ is (continuous or discrete) random variable. Some examples of uncertainty sources were already presented in the previous section.

The actual expenses of the arrangements are represented by the cost function $c: X \times \Xi \rightarrow R: (x; \xi) \mapsto c(x; \xi)$. It is nonlinear and can depend on the uncertainty variable. As we concentrate on different kinds of uncertainty factors in the problem, we make a simplification here and define $c(x; \xi) := c^T x$ (we drop the nonlinearity and explicit dependence of the costs on future uncertain factors); $c \in \mathbb{R}^n$ denotes a fixed (non-random) unit cost vector. Apart from a traditional cost-minimizing optimization problem, in our approach we incorporate the cost function to the constraint part of the problem; to do this, we suppose a constant $B$ representing a budget limit of the expenses. It is not hard to give back possible dependence into the inequality—we just proceed the same way as follows.

The factors of subjective and evaluative character are described by a utility function $u: X \times \Xi \rightarrow \mathbb{R}^m$. We analyze the utility function in more detail in the following section.

We are now ready to formulate an uncertain optimization problem in the form

$$\text{minimize } u(c; \xi) \text{ subject to } c^T x \leq B, x \in X_0.$$  \hspace{1cm} (1)

This formulation is more favorable to our view of the ecological policy as to obtain the best possible profit (utility). The only explicitly given constraints are represented by the cost constraint with budget limitation on the right-hand side. The set $X_0 \subset X$ covers for example 0-1 constraints, technical parameters or other deterministic constraints which are not explicitly specified here.

3 Stochastic Formulation of the Problem

The uncertain optimization problem (1) is not solvable in its present form due to the presence of uncertainty and the unknown form of the utility function. We will analyze both the issues in this section.

3.1 Indicators of Ecological Stability

As already noted above, the utility function is introduced in the context of representing a profit to the environment (and the humans) from ecological arrangements in large constructions which can be prescribed through the EIA process. The key idea of our approach is to replace a vaguely defined utility function by a set of well-defined quantities – indicators of ecological stability.

The indicators of ecological stability evaluate the quality of the environment in some specified area and are updated on a regular basis. The basic set is defined by the European Environmental Agency (EEA) Core Set of Indicators and includes such themes as air quality (for example: emissions of acidifying substances, emissions of ozone precursors,
etc.), biodiversity (number of bird species, protected areas, etc.), and many others. Some of the indicators (especially indicator of efficiency and total prosperity) are not yet standardized, and the number of proposed indicators grows as different studies make need of some not yet defined. These can be easily incorporated in our model if it would demonstrate the helpfulness.

Denote \( g: X \times \Xi \rightarrow \mathbb{R}^n \); \( (x; \xi) \mapsto g(x; \xi) \) a function representing the values of EEA indicators; \( g \) is a vector function with values in a space of a dimension corresponding to the number of indicators in questions. Having in hand medium or long-term time series of the indicators, we can estimate the distribution of the values and potentially also their dependence on specific decision \( x \). For example, the speed limits for vehicles on highways in urban areas have a provable influence on the overall noise level, and can be now even compared to historical datasets.

### 3.2 Weighting the Indicators

To replace the indeterminate utility function with indicators of ecological stability, it is still necessary to introduce a weighting scheme for these indicators. There are several possibilities to do this; in our paper we use a simple weighting approach based on a classical Allen’s indifference curves (Allen, 1934). The indifference curves model the levels of equal utility, and we also enable the possibility of to compensate a lack in one indicator by an improved value in another one. In this setting, the utility function reads as

\[
u(c; \xi) = w^T g(x; \xi)\]

where \( w \) is some prescribed weighting vector, representing indicator (ecological) preferences; \( w \) also compensates for different measurement scales of the indicators.

### 3.3 Introducing probabilistic constraints

It is usually required that the indicators of ecological stability satisfy several limits, often imposed by legislative standard. In our settings (allowing for compensations), we state only one such limit (denoted by \( L \) in the sequel) being an aggregate limit value for the function given by (2). This is necessary to reformulate the problem (1) in terms of probabilistic programming: instead of dealing directly with the objective function in form (2), we move the expression (2) into constraints by requiring

\[
w^T g(x; \xi) \geq L.\]

and maximizing the aggregate limit \( L \), which is now considered as another decision variable (maximum imposable limit). Of course, the last step is to explore stochastic nature of the parameter \( \xi \): we will require the constraint (3) to be satisfied with some, sufficiently high probability \( p \) (usually 0.95 or 0.99). The final optimization problem reads as

\[
\maximize L \text{ subject to } \Pr[w^T g(x; \xi) \geq L] \geq p, \ c^T x \leq B, \ x \in X_0.
\]

This problem is known as the optimization problem with a joint probabilistic constraint, see e. g. Prékopa (1995, 2003) for thorough investigation of this class of the problem, including theory and survey of numerical algorithms to solve such problems. In Houda (2011), we provide an extension to this probabilistic problem, replacing sometimes complicated probabilistic constraints with a simpler expectation functional in the objective function.

It is possible to prohibit compensations of indicator values. In this case, we have an individual limit \( L_j \) for each indicator value \( g_j(x; \xi) \) and require the indicator values to satisfy the limits jointly. To complete objective values we take a weighted sum of indicators—denote thus \( L := (L_1, \ldots, L_J)^T \) a vector of the indicator limits, and complete the model with the formulation

\[
\maximize w^T L \text{ subject to } \Pr[g_j(x; \xi) \geq L_j \forall j] \geq p, \ c^T x \leq B, \ x \in X_0.
\]

### 4 Results - Example

Suppose that the functional dependence of \( g \) on \( \xi \) is linear. In particular,

\[
g(x; \xi) \equiv x^T A \xi,
\]

the matrix \( A \) having deterministic coefficients. In Houda (2011), we have provided the following model. Suppose two indicators:

- percentage of area where the air pollution limits are exceeded, denoted by \( 1 - g_1 \), and
- number of habitants exposed to heavy noise, denoted by \( -g_2 \).
Furthermore, we suppose the dependence of both indicators on one uncertainty factor $\xi$, namely the random transport intensity on highway passing through the area in question. The possible arrangements are provided by

- imposed speed limit of 50 km/h (0-1 variable $x_1$),
- imposed speed limit of 80 km/h (0-1 variable $x_2$), and
- building noise wall of length $x_3$ (continuous variable).

The technical constraint $x_1 + x_2 \leq 1$ assures that only one of the speed limits offered is imposed. We assume the dependence of $g_1, g_2$ on $\xi$ to be linear, namely,

$$g_1(x; \xi) := (a_{10} + a_{11}x_1 + a_{12}x_2)\xi$$
$$g_2(x; \xi) := (a_{20} + a_{21}x_1 + a_{22}x_2 + a_{23}x_3)\xi$$

the coefficients $a_{ij}$ represents the positive effects of the imposed speed limits to the air and noise pollution, respectively. Introducing weighting coefficients $w_1, w_2$ for every of the two indicators, the problem formulation of type (5) then reads as

$$\text{maximize } w_1 L_1 + w_2 L_2$$
$$\text{subject to } \Pr \left( \begin{array}{c} (a_{10} + a_{11}x_1 + a_{12}x_2)\xi \geq L_1, \\
(a_{20} + a_{21}x_1 + a_{22}x_2 + a_{23}x_3)\xi \geq L_2 \\
c^T x \leq B, \, x_1 + x_2 \leq 1, \, x_1, x_2 \in \{0,1\}, \, x_3 \geq 0 \right) \geq p.$$ 

The resulting optimization program is the linear stochastic programming mixed-integer problem with probabilistic constraints. If, in addition, we provide some additional assumptions on distribution of the random variable $\xi$ (for example its normality), it is numerically solvable (see e. g. Cheng & Lisser, 2012; Houda & Lisser, 2014). In this paper, we will not go into the details of numerical computations of probabilistically constrained optimization problems – see the references above.

5 Conclusions

During the process of the Environmental Impact Assessment (EIA), various ecological measures are considered in order to discover the impacts of big industrial and transport constructions to the environment, and to provide recommendations to the investors to adjust the projects to meet the ecological requirements. Many of these measures depend on factors which are uncertain (random) by nature. We propose a methodology which deals with these uncertain factors; in particular, we formulate two stochastic programming problems which deal with the task to introduce ecological arrangements of the construction in order to diminish the impact of the construction to the environment.

Both proposed models (4), (5) follow our aim: to maximize the overall utility from introducing ecological arrangements, provided the total costs will not exceed the budget limit. The uncertainty is caught through a probability distribution of the uncertainty vector $\xi$ and the constraints are required to be satisfied with sufficiently high probability. Under additional assumptions on the probability distribution of the random component of the problem, the proposed models are solvable with linear or nonlinear algorithms introduced in the literature.

References