METHODS FOR SOLVING THE VEHICLE ROUTING PROBLEM

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Abstract

The traveling salesman problem (TSP) belongs to the NP-hard problems. It means that there exist no methods, which would be able to compute its theoretical optimum efficiently. Thus there exist a lot of heuristics (approximation methods), which give only approximate solutions but considered to be economic optima.

Vehicle routing problems (VRP) are related to the TSP. They describe situations when it is not possible to carry out the transportation using one cyclic route (e.g. because of capacity or time reasons) and so it is necessary to create more than one such route. The reasons why this situation happens can be different. Therefore, single instances of the VRP are very different, too, and so researchers are much less concentrated on the VRP than on the TSP. In this contribution selected methods for the TSP are reviewed according to the possibility and suitability of their modification for the VRP. Some test results of several methods created in such a way are added.

Key words: vehicle routing problem, heuristics (approximation methods).

Introduction

The traveling salesman problem (TSP) is a task defined as follows: we are given \( n \) cities (places, points) and a distance (cost for transportation) for each pair of them and we are to find a cyclic route (Hamiltonian cycle) of minimum possible length passing exactly once through each of these cities. This task belongs to the NP-hard problems, for which there is no efficient algorithm finding their theoretical optimum. So the only way how to obtain efficiently or in a reasonably short time some solution is to use some of heuristics (approximation methods), which give only “good” or “close to optimal” solution, not exactly optimum.

In practice we often meet more complicated situations when it is not sufficient to use only one circular tour. These tasks are called vehicle routing problems (VRP). One of the most common cases when this situation happens is when we need to distribute certain material from (or to) the central point to (or from) finite number of places and the capacity of one vehicle is not sufficient. Thus we have to design more than one circular tour, use more than one vehicle or the vehicle must make more tours. Similar situation occurs in passenger transport where we have to take in mind not only the vehicle capacity but also a reasonable duration of one tour. Such a problem we call multipletours traveling salesman problem (MTTSP).
Literary overview

The TSP is a problem, which is solved often, and so there are many heuristic methods for it. The VRP occurs perhaps more often, but the reasons why the transportation cannot be realized using one cyclic tour can be different and so it is difficult to design a method usable for at least slightly wide class of VRPs. So the amount and choice of different heuristics for single types of VRP including the MTTSP are smaller than for the TSP.

A detailed overview of various methods for the TSP and related problems is in (3), which has recently appeared. References to the papers where single methods have been first published are appended to all of them in the following text, except for the Mayer method, which is proposed by a Czech author and it is not published in any outstanding paper or proceedings (at least the author of this contribution has not managed to find any reference). A precise description of versions of these methods for the MTTSP can be found in (9).

Material and methods

The aim of this contribution is to consider, which heuristics for the TSP are suitable to be modified for the MTTSP and which are not. Each method is described here first in the original version for the TSP and then follow either its modification for the MTTSP or reasons why it is not suitable for the MTTSP. In the end some results of tests of these methods for the MTTSP on several instances taken from (9) are added.

In this contribution we will restrict us only for the tasks with the symmetric cost matrix.

Results and discussion

There exist a lot of heuristics for the TSP and so their list cannot be complete. The tree approach, the savings method and the Habr frequencies approach represent the methods, which can be modified for the MTTSP, while the patching method, the loss method and the exchange methods are not suitable for this purpose.

Tree approach

The tree approach for the MTTSP is known as the Mayer method. Its procedure is relatively different from the way, how to utilize trees for solving the TSP. Thus only the version for the MTTSP will be described here. Let us remark that this method only separates cities into groups so that each group contains cities on a route for one vehicle and then some of the methods for the TSP must be used to determine the order of the cities on the single cycles.

Mayer method creates these groups one by one. First the remotest city from the central one is put into a new group. Then the nearest city to some of the cities already being in the group is always added. So the method works similarly like the Prim (Jarník) algorithm for the minimum spanning tree (12, 6). The group is complete if adding the next city the capacity of the vehicle is exceeded.

Savings method

This method was designed by Clarke and Wright (1), even for the VRP, too. At first one city is chosen, let us denote it 0. Then for each pair of the other cities the savings between the route via city 0 and the straight route are computed. These pairs (straight routes) are ordered according to its savings and then they are consequently processed and added into the solution
(resulting circular route) in the case that they can form the circular route with already created part of the solution. In the end the city 0 is added.

For the MTTSP the central city is taken as the city 0. During the creation of the solution it is necessary to watch if the sums of capacities of the cities on single paths (connected parts of the solution) do not exceed the capacity of the vehicle. Again, in the end the city 0 is added.

Habr frequencies approach

The disadvantage of savings is that they compare a given edge (straight route between two cities) with only one route via only one city chosen for all the computation. Habr (4, 5) introduced so called frequencies, which compare the edge with all the others, even non-adjacent edges. He applied them in approximation methods for different transportation problems and designed even several methods how to use them for solving the TSP. The exact formula, from which the sense of the frequencies is obvious, is rather complicated and thus it is not presented here (cf. 9), but it can be simplified for computation purposes.

Habr frequencies consider all edges with the same importance. But in the case of MTTSP the edges from/to the central city are more important (more frequently and often used) than the others. Thus it is suitable to modify the frequencies formulas to consider the central city with bigger importance than others. Using the frequencies improved by this way the rest of the computation is similar to the savings method, i.e. the pairs of cities are ordered according to the frequencies and processed in this order to obtain the solution in the same way like in the savings method.

Patching method

Karp (7) proposed a method based on connecting (patching) cycles in the solution of the assignment problem with the same cost matrix, which the solved instance of the TSP has. First the assignment problem is solved using the Hungarian method. Then two longest cycles are taken and connected into one cycle so that the increment of the objective function is as minimal as possible. In this way the number of cycles contained in the solution decreases by one. Repeating this procedure a solution consisting of only one cycle (i.e. a feasible solution of the TSP) is obtained in the end.

In the case of the task with symmetric cost matrix the optimum assignment consists of cycles of length (number of cities) two except for one cycle of length three in the case of odd number of cities. Applying it for the MTTSP the routes for single vehicles would contain even number of other cities except the central one (except for one route in the case of odd number of the non-central cities). This is a significant restriction of the set of available solutions and so this method is not suitable for modifying for the MTTSP.

Loss method

This approach is usable for different distribution tasks. It was modified for the TSP by Webb (14) and Van der Cruyssen and Rijckaert (13). For all lines (rows and columns) in the cost matrix the differences between two lowest (best) costs are computed. In the line with the biggest difference the lowest cost is used (the corresponding edge is added to the solution). All such costs the corresponding edges of which cannot form with already created part a feasible solution (circular route) are omitted for the rest of the computation and the differences are recomputed. The procedure is repeated until the circular route is obtained.

Modifying this method for the MTTSP it is difficult to find all the costs (edges), which cannot be added to the created solution in respect of the capacities. Thus this method is not suitable for the MTTSP.
Exchange methods

These methods take an initial solution obtained either randomly or using some of other heuristic method and try to improve it replacing several its edges by the same number of other edges so that the circular route arises again but with a smaller value of the objective function. The first one, who came with this idea, was Croes (2). Several such methods were designed by Lin (10), the most famous one in his common work with Kernighan (11). Single methods differ in the technique how to specify the edges to exchange.

These methods are not usable for the MTTSP because by exchanging edges from routes of different vehicles these routes are connected into one route. Because the routes for single vehicles are usually short, exchanges in one route do not help much and, in addition, it is actually a matter of solving the TSP for a particular vehicle.

Test results

For testing, the randomly generated cases described in (8) were taken. Let us remind that the central city was located in the middle of the attended region where there were 12 other cities.

Five cases of the MTTSP were solved using MS-Excel by all three methods for the MTTSP described above. The results are summarized in the percentage form, where 100 p.c. is the result of the Mayer method, in the table 1.

<table>
<thead>
<tr>
<th></th>
<th>Mayer</th>
<th>Savings</th>
<th>Habr Freq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>100,0%</td>
<td>90,9%</td>
<td>90,9%</td>
</tr>
<tr>
<td>Case 2</td>
<td>100,0%</td>
<td>98,8%</td>
<td>100,7%</td>
</tr>
<tr>
<td>Case 3</td>
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<td>100,5%</td>
<td>100,5%</td>
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<td>100,0%</td>
<td>97,9%</td>
<td>93,5%</td>
</tr>
<tr>
<td>Case 5</td>
<td>100,0%</td>
<td>91,6%</td>
<td>93,1%</td>
</tr>
</tbody>
</table>

Table 1 – Test cases results

Conclusion

Among the methods considered here the tree approach, the savings method and the Habr frequencies approach are suitable for modification and application for the (symmetric) MTTSP, while the patching method, the loss method and the exchange methods are not suitable. Let us summarize advantages and disadvantages of the single methods the application of which for the MTTSP is possible.

The Mayer method is the simplest one from the methods for the MTTSP described here. Perhaps, it is the reason why it is the most popular one although it does not solve the task completely. It only divides cities into groups for single routes (vehicles) and then it is necessary to solve TSPs for each vehicle. Nevertheless, it does not seem to be an important complication, especially in the cases when short cycles with only a few cities are expected.

The savings method and the Habr frequencies approach have several advantages. They solve the MTTSP completely and it is not necessary to solve any other, though small, additional problems. In the most of the test cases solved here they found better solution than the Mayer method.

Another interesting property is that all these three methods almost always gave different solutions for a given instance. Therefore, for solving a task in practice it is suitable to try all these methods and choose the best solution between all obtained in this way.

Of course, for the solid analysis of their results the methods are to be tested on different types of test instances.
References

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